OSCILLATING FLOW IN A TUBE WITH FORMATION OF A BOUNDARY LAYER OF SOLVENT*

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It is shown that the presence of a boundary layer of solvent can lead to a sharp increase in the velocity of an oscillating flow of viscoelastic fluid.

Interest in oscillating flows has greatly increased in recent years. This is due to their diverse practical applications [1]. In particular, they are widely used in experimental methods of determining the parameters of models of viscoelastic fluids [2, 3].

Oscillating flows of an isotropic Newtonian fluid [4, 5] a non-Newtonian inelastic fluid [6, 7], and a viscoelastic fluid [2, 3] have now been theoretically analyzed. In the case of a flow of emulsions, suspensions, and some polymer solutions a layer with properties similar to those of the solvent can be formed in the immediate vicinity of a pipe step [8, 9]. It has been shown experimentally [10] that an oscillating flow promotes the formation of such a layer. In the wall region a viscosity considerably less than the cited viscosity can be obtained as a result of thixotropic effects.

The aim of the present work was to quantitatively estimate the effect of such a boundary layer on the flow velocity profile.

We assume that a periodic pressure drop in a round tube of radius R causes a laminar flow of a viscoelastic fluid whose behavior is characterized by the Oldroyd model [11]

$$\tau = \mu_0 (\dot{\gamma} + \lambda_2 \ddot{\gamma}) - \lambda_1 \dot{\tau}. \tag{1}$$

This model is applicable to emulsions and some polymer solutions [12].

In the wall region there is formed a layer of Newtonian fluid of thickness $\delta \ll R$, which may be a disperse medium (suspension, emulsion) or the solvent of a polymer solution. For such a layer we can use the relation

$$\tau = \mu \dot{\gamma}. \tag{2}$$

Using the complex representation of a periodic process we can write the following equations for the pressure gradient, velocity, and deformation stress sufficiently far from the tube entrance [13]

$$p = p_0 \exp(i\omega t), \; W = w \exp(i\omega t), \; \tau = \tau_0 \exp(i\omega t). \tag{3}$$

The amplitudes w, p_0, and τ_0 in the most general case are complex numbers containing an appropriate phase angle.

The subscript 1 denotes quantities relating to the boundary layer and the subscript 2 denotes those for the main fluid. Substitution of Eqs. (1) and (2) in the equation of motion in cylindrical coordinates

$$\rho \frac{\partial W_z}{\partial t} = -\frac{\partial P}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r}(r\tau_{rz}) \tag{4}$$

after appropriate algebra leads to the system

$$\frac{d^2 w_1}{dr^2} + \frac{1}{r}\frac{dw_1}{dr} + \beta^2 w_1 = \frac{p_0 \beta^2}{i\omega\rho_1}, \tag{5}$$

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$$\frac{d^2w_2}{dr^2}+\frac{1}{r}\frac{dw_2}{dr}+\lambda^2 w_2=\frac{p_0\lambda^2}{i\omega\rho_2}, \qquad (6)$$

where the parameters β and λ are given by the expressions

$$\beta^2=-\frac{i\omega\rho_1}{\mu}, \quad \lambda^2=-\frac{i\omega\rho_2(1+i\omega\lambda_1)}{\mu_0(1+i\omega\lambda_2)}. \qquad (7)$$

Equations (5) and (6) are valid for a number of other models of a viscoelastic fluid. For λ^2 we obtain the corresponding expressions given in [3].

The substitution $w = v + p_0/i\omega\rho$ converts Eqs. (5) and (6) to a system of differential Bessel equations:

$$\frac{d^2v_1}{dr^2}+\frac{1}{r}\frac{dv_1}{dr}+\beta^2 v_1=0, \qquad (8)$$

$$\frac{d^2v_2}{dr^2}+\frac{1}{r}\frac{dv_2}{dr}+\lambda^2 v_2=0. \qquad (9)$$

Their general solutions have the form

$$v_1=A_1J_0(\beta r)+B_1Y_0(\beta r), \qquad (10)$$

$$v_2=A_2J_0(\lambda r)+B_2Y_0(\lambda r), \qquad (11)$$

where J_0 and Y_0 are zero-order Bessel functions of the first and second kinds, respectively. Coefficients A_1, B_1, A_2, and B_2 are given by the following boundary conditions:

$$\text{a) } r=R,\ w_1(r)=0;\ \text{c) } r=0,\ w_2 - \text{constant}; \qquad (12)$$

$$\text{b) } r=R-\delta,\ w_1(r)=w_\delta;\ \text{d) } r=R-\delta,\ w_2(r)=w_\delta,$$

where w_δ is the velocity amplitude at the boundary of the boundary layer and the main fluid.

Boundary condition c) requires that $B_2 = 0$, and boundary condition d) leads to the equality

$$A_2=\frac{w_\delta}{J_0[\lambda(R-\delta)]}-\frac{p_0}{i\omega\rho_2 J_0[\lambda(R-\delta)]}. \qquad (13)$$

Then the distribution of the velocity amplitude of the main fluid across the tube is given by the equation

$$w_2=\frac{p_0}{i\omega\rho_2}\left\{1-\frac{J_0(\lambda r)}{J_0[\lambda(R-\delta)]}\right\}+\frac{w_\delta J_0(\lambda r)}{J_0[\lambda(R-\delta)]}. \qquad (14)$$

Boundary conditions a) and b) give formulas for coefficients A_1 and B_1:

$$A_1=\frac{w_\delta i\omega\rho_1 Y_0(\beta R)+p_0\{Y_0[\beta(R-\delta)]-Y_0(\beta R)\}}{i\omega\rho_1\{J_0[\beta(R-\delta)]Y_0(\beta R)-Y_0[\beta(R-\delta)]J_0(\beta R)\}}, \qquad (15)$$

$$B_1=-\frac{p_0}{i\omega\rho_1 Y_0(\beta R)}-\frac{J_0(\beta R)\{w_\delta i\omega\rho_1 Y_0(\beta R)+p_0\{Y_0[\beta(R-\delta)]-Y_0(\beta R)\}\}}{i\omega\rho_1 Y_0(\beta R)\{J_0[\beta(R-\delta)]Y_0(\beta R)-Y_0[\beta(R-\delta)]J_0(\beta R)\}}. \qquad (16)$$

The velocity amplitude profile in the boundary layer is

$$w_1=\frac{p_0}{i\omega\rho_1}+A_1J_0(\beta r)+B_1Y_0(\beta r). \qquad (17)$$

Differentiating (14) and (17) we obtain the velocity gradients for the two layers:

$$\frac{dW_2}{dr}=\left\{\frac{p_0\lambda J_1(\lambda r)}{i\omega\rho_2 J_0[\lambda(R-\delta)]}-\frac{w_\delta\lambda J_1(\lambda r)}{J_0[\lambda(R-\delta)]}\right\}\exp(i\omega t), \tag{18}$$

$$\frac{dW_1}{dr}=-\exp(i\omega t)\{A_1\beta J_1(\beta r)+B_1\beta Y_1(\beta r)\}. \tag{19}$$

From the condition for equality of stress at the boundary of the two regions

$$\frac{\mu_0(1+i\omega\lambda_2)}{1+i\omega\lambda_1}\left(\frac{dW_2}{dr}\right)_{r=R-\delta}=\mu\left(\frac{dW_1}{dr}\right)_{r=R-\delta} \tag{20}$$

we obtain the velocity w_δ required for calculation of the velocity profile from Eqs. (14) and (17):

$$w_\delta=C/D; \tag{21}$$

the values of C and D are given in the Appendix.

Tables [14, 15] give the values of the Bessel functions correct to the eighth decimal place for the first quadrant of the complex region. This accuracy is particularly important in calculation of the difference

$$J_0[\beta(R-\delta)]\,Y_0(\beta R)-Y_0[\beta(R-\delta)]\,J_0(\beta R)$$

in expressions C and D.

Functions $J_0(\lambda r)$ and $J_1(\lambda r)$ are expressed with the aid of tabulated real (u_0, u_1) and imaginary (v_0, v_1) parts

$$J(\lambda r)=J(\bar{r},\ \exp i\varphi)=u(\bar{r},\ \varphi)+iv(\bar{r},\ \varphi), \tag{22}$$

where $\bar{r}$ and φ are found from the equations in [3].

Functions $J_0(\beta r)$ and $J_1(\beta r)$ are calculated with the aid of tabulated functions ber and bei [16] or tables [14] from the equations:

$$J_0(\beta r)=u_0(\bar{r},\ \pi/4)-iv_0(\bar{r},\ \pi/4), \tag{23}$$

$$J_1(\beta r)=-u_1(\bar{r},\ \pi/4)+iv_1(\bar{r},\ \pi/4), \tag{24}$$

and functions $Y_0(\beta r)$ and $Y_1(\beta r)$ from

$$Y_0(\beta r)=\{U_0(\bar{r},\ \pi/4)+2v_0(\bar{r},\ \pi/4)\}-i\{V_0(\bar{r},\ \pi/4)-2u_0(\bar{r},\ \pi/4)\}, \tag{25}$$

$$Y_1(\beta r)=\{-U_1(\bar{r},\ \pi/4)-2v_1(\bar{r},\ \pi/4)\}+i\{V_1(\bar{r},\ \pi/4)-2u_1(\bar{r},\ \pi/4)\}, \tag{26}$$

where u and v are taken from [14], and U and V from [15].

In Eqs. (23)-(26) $\bar{r}=r\sqrt{\omega\rho_1/\mu}$.

If there is no boundary layer ($\delta = 0$, $w_\delta = 0$), then (14) reduces to the form given in [2] and [3].

Calculations made from Eqs. (14) and (17) showed that the presence of a boundary layer can lead to a sharp increase in the velocity in the wall region even if the layer is very thin. Directly at the wall the effect is probably manifested as wall slip, and is more pronounced than for steady-state laminar flow of the same fluid. This means that an oscillating flow can provide the basis of a very sensitive experimental technique for determination of the existence of a boundary layer.

Increase in layer thickness enhances the "slip" effect, which leads to an increase in the mean displacement at the particular pressure gradient. If the mean displacement is kept constant the energy spent on generation of the oscillating flow will be reduced.

Figure 1 illustrates what we have said above for the following conditions: $\lambda_1 = 0.456$ sec, $\lambda_2 = 0.3$ sec, $\omega = 5\ sec^{-1}$, $\rho_1 = \rho_2 = 1125\ kg/m^3$, $\mu_0 = 0.1243$ Pa•sec, $\mu = 10^{-2}$ Pa•sec, R = 0.012 m.

In [10] boundary layers of thickness $2.5\cdot10^{-4}$–$5\cdot10^{-4}$ m were observed in an oscillating flow of suspension.

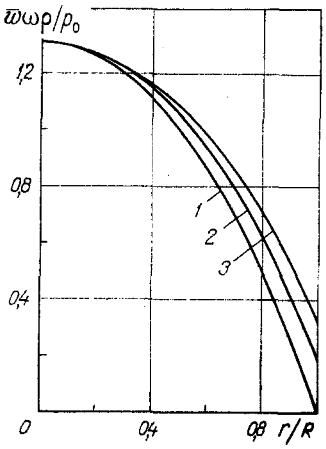

Fig. 1

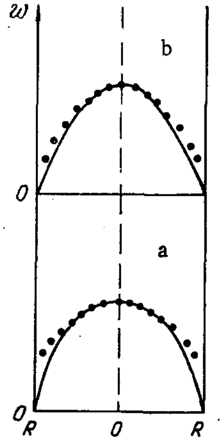

Fig. 2

Fig. 1. Velocity amplitude profile for different boundary layer thicknesses: 1) $\delta = 0$; 2) 0.00012 m; 3) 0.00024 m.

Fig. 2. Comparison of experimental data of [7] with theoretical solutions of same authors. Aqueous solution of carboxymethylcellulose with concentration 0.5% (a) and 1% (b) and oscillation frequency 1 Hz.

Figure 1 shows that the effect of the boundary layer becomes negligible in the axial region of the tube. This confirms the earlier recommendation [3] that the velocity profiles in the central part of the tube should be used, since they are fairly sensitive and are not subject to the wall effect.

The formation of a boundary layer would account for the much higher velocities observed in the wall region for an oscillating flow of aqueous solutions of carboxymethylcellulose [7] in comparison with the theoretical values for an isotropic medium (Fig. 2). Such solutions showed other anomalies, such as drag reduction [17], accelerated runoff of the film [18], and other features which could be attributed to the presence of a boundary layer of solvent.

Failure to consider the actual velocity profile can lead to erroneous estimates of the effect of oscillation of the fluid on heat and mass transfer.

APPENDIX

$$C = \frac{p_0 Y_1[\beta(R-\delta)]}{\beta Y_0(\beta R)} - \frac{p_0 J_1[\lambda(R-\delta)]}{\lambda J_0[\lambda(R-\delta)]} -$$

$$-\frac{p_0\{J_1[\beta(R-\delta)]Y_0(\beta R) - Y_1[\beta(R-\delta)]J_0(\beta R)\}\{Y_0[\beta(R-\delta)] - Y_0(\beta R)\}}{\beta Y_0(\beta R)\{J_0[\beta(R-\delta)]Y_0(\beta R) - Y_0[\beta(R-\delta)]J_0(\beta R)\}},$$

$$D = \frac{\mu_0\lambda(1+i\omega\lambda_2)J_1[\lambda(R-\delta)]}{(1+i\omega\lambda_1)J_0[\lambda(R-\delta)]}$$

$$+\frac{\mu\beta\{Y_1[\beta(R-\delta)]J_0(\beta R) - J_1[\beta(R-\delta)]Y_0(\beta R)\}}{J_0[\beta(R-\delta)]Y_0(\beta R) - Y_0[\beta(R-\delta)]J_0(\beta R)}.$$

NOTATION

ω, frequency of oscillating flow, sec^{-1}; ρ, fluid density, kg/m^3; λ_1, relaxation time, sec; λ_2, delay, sec; μ_0, viscosity at infinitely low deformation rate, Pa•sec; τ, shear stress, Pa; $\dot{\gamma}$, deformation rate, sec^{-1}; R, tube radius, m; δ, thickness of boundary layer, m; p, pressure gradient, Pa/m; W, fluid velocity, m/sec; t, time, sec; $\bar{w}$, modulus of velocity amplitude, m/sec.

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THERMAL CONDUCTIVITY OF FREON-218

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The thermal conductivity of Freon-218 is investigated experimentally in a wide region of the parameters. Reference tables of thermal conductivity are compiled.

Freon-218 (C_3F_8) is a promising agent for refrigeration and especially for cryogenic engineering, but its application is limited by the absence of data on the thermophysical properties in the region of the parameters required in practice. Earlier we [1] determined the thermal conductivity of Freon-218 at low temperatures (from 113 to 297°K). The aim of the present report is an investigation of the thermal conductivity of Freon-218 at moderate and moderately high temperatures (up to 430°K) and pressures up to 60 MPa, as well as the development of reference tables of λ.

The thermal conductivity was measured by the hot-filament method using a cell whose construction is described in [2]. In all the tests λ was determined at different temperature drops in the layer, with the Rayleigh numbers not exceeding 1500. The region of the maxima (at $0.6 < \omega < 1.4$ and $\tau < 1.15$) was not investigated. The experimental results are presented in Table 1.

In the treatment of the measurement results we analyzed the equations

$$\lambda - \lambda_t = \sum_{i=1}^{n} \sum_{j=0}^{s_i} a_{ij}\omega^i/\tau^j, \tag{1}$$

$$\ln(\lambda/\lambda_t) = \sum_{i=1}^{n} \sum_{j=0}^{s_i} a_{ij}\omega^i/\tau^j. \tag{2}$$

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